



A DISCRETE MODEL FOR GROWTH, SURVIVAL AND FOOD CONSUMPTION OF AMERICAN BULLFROG: A MANAGEMENT-ORIENTED SCOPE

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ABSTRACT

An algebraic model in discrete time was developed to assess the food budget in frog farms. Three main functions were taken into account to determine average daily food intake: mean individual weight, number of individuals and daily food ration. A calculation example was carried out, taken into account four different simulation cases. Two of them used algebraic growth functions based in previous experimental work, and the other two assumed simple exponential growth. For all cases, an exponential decay rate of surveillance was used, and all of them consider the daily feeding ration function proposed by Lopes-Lima & Agostinho (1998). The model reproduces the feeding schedule used by Rodríguez-Serna et al. (1996), and simulates feeding budget derived of the exponential growth processes. Compared to previous experimental results, the calculated relative error on final biomass estimations were lower than 1.23 %, meanwhile relative error in annual estimated productivity was lower than 3.07%. The model can be easily implemented in non-specialized software to simulate different scenarios, and it could be an useful tool to make policies and decisions in frog farm managing.

Keywords Aquaculture, Food consumption, *Rana catesbeiana*, Farm management, Discrete mathematical model

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INTRODUCTION

American Bullfrog (*Rana catesbeiana*) is, by far, the most commercially important anuran species. Its healthy meat, combined with specific and valuable by-products are the main reasons of continuous rise in frog production since 1980's (Fenerick & Verardino, 2005). For example: in European countries, frog legs are a valuable product in specific culinary styles; in medicine, frog-gut is used as suture; and in education field, frogs are usually required for zoological dissections (Neveu, 2009; Olvera-Novoa et al., 2007; Martínez



In intensive frog production systems, like other aquacultural facilities, is essential to make good predictions for the assessment of adequate management practices (Alatorre-Jácome *et al.* , 2011). Two main goals to achieve economical success in every frog farm are: 1), to maximize biomass production; and 2), to minimize operational costs. To predict production performance under different scenarios, the use of mathematical models based under well-known assumptions could be useful. In this case, the modeler can be able to obtain reliable knowledge of the system (For a extensively discussion and examples, see Thornley & France (2006)).

In literature or practice, is usual to conduct experiments to obtain specific information about individual or collective growth performance. The obtained data can be used to fit simple algebraic curves using linear regressions techniques (See Bequette (1998), appendix 3). For example, Yilmaz *et al.* (2005) obtained the relationship between length and age (in males), obtaining the equation $y = 77.04 - 32.53e^{-0.49(x-1)}$. Two years later, Miaud *et al.* (2007) modeled growth of *Rana holtzi* in Turkey by the equation $BL_t = BL_{max} - (BL_{max} - BL_{mete})e^{-k(t-t_{mete})}$; where t is the age in years, BL_t is the total length at age t , and BL_{max} is the estimated average maximum total length at metamorphosis. More recently, de Seixas *et al.* (2010) modeled tadpole growth in the quadratic equation $y = 0.0457 - 0.005017x + 0.0032x^2$. These equations are valuable due to their application estimated total frog biomass in the farm under experimental conditions if the total number of frogs is known, or if can be estimated. In this case, the farm manager can design feeding schedules or run mathematical simulation to achieve that goal.

The aim of the present work was to construct a discrete, algebraic food budget model based on two main assumptions: One, the growth function of the mean individual weight can be described by previous experimental data fitting; and two, the surveillance of the frogs in the same time describes a negative exponential curve. During the culture time, the daily feed ration can be selected by the manager or can be expressed as a function of the individual weight. Due to the discrete and algebraic conception of the model, it can be easily implemented in many computational packages, and its results can be used to make quick assessments in farm managing.

Theory

The general structure of the model begins with a time-dependent growth function:

$$\bar{W} = f(t), \dots \quad (1)$$

where \bar{W} is the estimated individual body weight of the frog and $f(t)$ is a time-dependent function. If we consider a discrete algebraic function, we can rewrite eq. 1 as:-

$$\bar{W}_t = f_t. \dots \quad (2)$$

Many growth functions, both theoretical or empirical, can be used to estimate biological growth (see Thornley & France (2006), chapter 5). If the manager can estimate individual body weight by a reliable function $W^* t$, then the estimated total frog biomass, B_t , can be calculated by (Alagaraja,1991):

$$B_t = N_t \bar{W}_t, \dots \quad (3)$$

where N_t is the total number of individuals in time t . If we assume only natural mortality in



the farm, the number of frogs in the instant t , can be calculated by a negative exponential equation (Sparre & Venema, 1998):

$$N_t = N_0 e^{Zt} \dots \dots \dots (4)$$

where N_0 is the initial number of frogs in the farm and Z is the instantaneous rate of total mortality, calculated by:

$$Z = - \left[\frac{\ln(S_f) - \ln(S_i)}{t_f} \right] \dots \dots \dots (5)$$

where S_i and S_f is the fraction of living frogs in an initial and final time, respectively; and t_f is the final time. When the values of mortality change over the time, the values of Z are:

$$Z = \begin{cases} a_z & \text{if } t_i < t \leq t_{a_z} \\ b_z & \text{if } t_{a_z} < t \leq t_{b_z} \\ c_z & \text{if } t_{b_z} < t \leq t_{c_z} \\ \vdots & \vdots \quad \vdots \\ n_z & \text{if } t_{n_z} < t \leq t_f \end{cases} \dots \quad (6)$$

In this case, the values of N_o and t in equation 6 must be reset. For example, in every new case, a new value of N_o corresponds to $N_o = N_{t-1}$, and the new value of t begins in 1. The daily feed input, F_t , can be calculated by:

where ft is the feed fraction proportional to W . In many cases, this fraction is determined by the farm manager, and it depends both on the average frog wet weight and the time t . A general case can be represented by the following function:

$$f_t = \begin{cases} a_{f_t} & \text{if } w_i \leq \bar{W}_t \leq w_{a_{f_t}} \\ b_{f_t} & \text{if } w_{a_{f_t}} < \bar{W}_t \leq w_{b_{f_t}} \\ c_{f_t} & \text{if } w_{b_{f_t}} < \bar{W}_t \leq w_{c_{f_t}} \\ \vdots & \vdots \quad \vdots \\ n_{f_t} & \text{if } w_{n_{f_t}} < \bar{W}_t \leq w_f. \end{cases} \dots \quad (8)$$

Thus, to evaluate the estimated feed consumption (FC) in time t , the general expression is:

To calculate the total feed weight (TFW) in time t, we use the following equation:



$$TFW_t = \sum_{t_i=0}^t FC_t \quad \dots \dots \dots \quad (10)$$

The feed conversion ratio (FCR) in time t is given by:

$$FCR_t = \frac{TFW_t}{B_t} \quad (11)$$

The relative density (RD) in time t is given by

$$RD_t = \frac{N_t}{A} \quad (12)$$

Where A is the culture area, usually measured in m^2

Calculation

Particular cases

The simulation was carried out by taken into account the previous experimental work of Rodríguez-Serna et al. (1996). These authors validated the performance of a vertical intensive culture system under three different stoking densities (50, 100 and 200 frogs m⁻²). During 175 days of experimentation, the production of the first two treatments was evaluated. Growth measures and productive indexes were calculated at the end of the experiment (Table 1). Two of them uses the empirical equations proposed by Rodríguez-Serna et al. (1996) obtained in frogs cultured using two different stoking densities (50 and 100 frogs m⁻², \bar{W}_{1t} and \bar{W}_{2t} , respectively). The other two functions (\bar{W}_{3t} and \bar{W}_{4t}) uses a theoretical simple exponential growth model (Table 2). To calculate the frog abundance, the equations used in the simulation can be seen on Table 3. To determine food consumption, only one function was used according to the recommendations of Lopes-Lima & Agostinho (1998). The function can be seen on equation 13:

$$f_t = \begin{cases} 0.12 & \text{if } 2 < \bar{W}_t \leq 10 \\ 0.11 & \text{if } 10 < \bar{W}_t \leq 20 \\ 0.10 & \text{if } 20 < \bar{W}_t \leq 30 \\ 0.09 & \text{if } 30 < \bar{W}_t \leq 50 \\ 0.08 & \text{if } 50 < \bar{W}_t \leq 65 \\ 0.07 & \text{if } 65 < \bar{W}_t \leq 90 \\ 0.06 & \text{if } 90 < \bar{W}_t \leq 110 \\ 0.05 & \text{if } 110 < \bar{W}_t \leq 150 \\ \leq 0.05 & \text{if } \bar{W}_t > 150. \end{cases} \dots \quad (13)$$

Thus, the four particular simulation cases are generated sustaining the values of the growth and abundance functions in equation 9 (Table 4).



Model implementation and validation

For the model implementation, both calculations and graphics were developed using Excel© datasheets. Because the model replicates feeding schedule followed by Rodríguez-Serna et al. (1996), and empirical equations used in case 1 and case 2 showed a high correlation coefficients with actual experimental data ($r^2 = 99.53$, for case 1; and $r^2 = 99.79$ for case 2), we assume these cases as the experimental procedure.

Results

Numerical simulations were carried out to estimate mean individual weight, total abundance, biomass, relative density, food ration, daily food weight, cumulative food, and feed conversion ration (Fig. 1). Productivity parameters also were calculated (Table 5). For the individual mean estimated weight, case 1 and case 2 showed a faster growing rate in the first half of the modeling time (Figure 1a). However, during the second modeling half (after 90 days), the growing rate increased faster in case 3 and 4. Rodríguez-Serna et al. (1996) reported two different survival rates in the experiment: the first measured until week 8 (81.3 % in 50 frogs m⁻²; and 79.3 % in 100 frogs m⁻²); and the final survival rate (72 % in 50 frogs m⁻²; and 65 % in 100 frogs m⁻²). However, in this study only the final mortality was taken into account. When surveillance (Fig. 1b) and relative abundance (Fig. 1d) were simulated, the performance showed almost a linear decay rate. The two equations predicted exact values in the beginning and in the end of the simulations, compared with actual reported data.

For the biomass estimation, the effects of the abundance enhance the performance differences between the two densities. Case 1 and 2 present an expo-linear dynamic performance, meanwhile case 3 and case 4 remain a typical exponential growth (Fig. 1c). Despite the differences in the dynamics, the two different approaches estimated closest values in final biomass (Table 5). The absolute difference between biomass estimations in case 1 and case 3 is 0.155 g, and the absolute difference in case 2 and 4 is 0.332 g.

The daily food intake decreased faster in case 1 and 2 than in case 3 and 4 (Fig. 1e). For example, in case 1 and case 2, food intake begins to change in days 36 and 37 respectively, whereas in case 3 and case 4 the same shift occurs until day 67 and 71. This represents almost the double of time to reach individual weight of 10 grams.

The geometric representation of the total estimated daily food ration is, in all cases, a non-smooth graphic (Fig. 1f). In case 1, the performance is almost linear until day 108, when the food ration changed from 0.09 to 0.08. Similar results can be observed in case 2, from the beginning of the simulation until day 114. In cases 3 and 4, the tendency is almost exponential, except when food ration shifts.

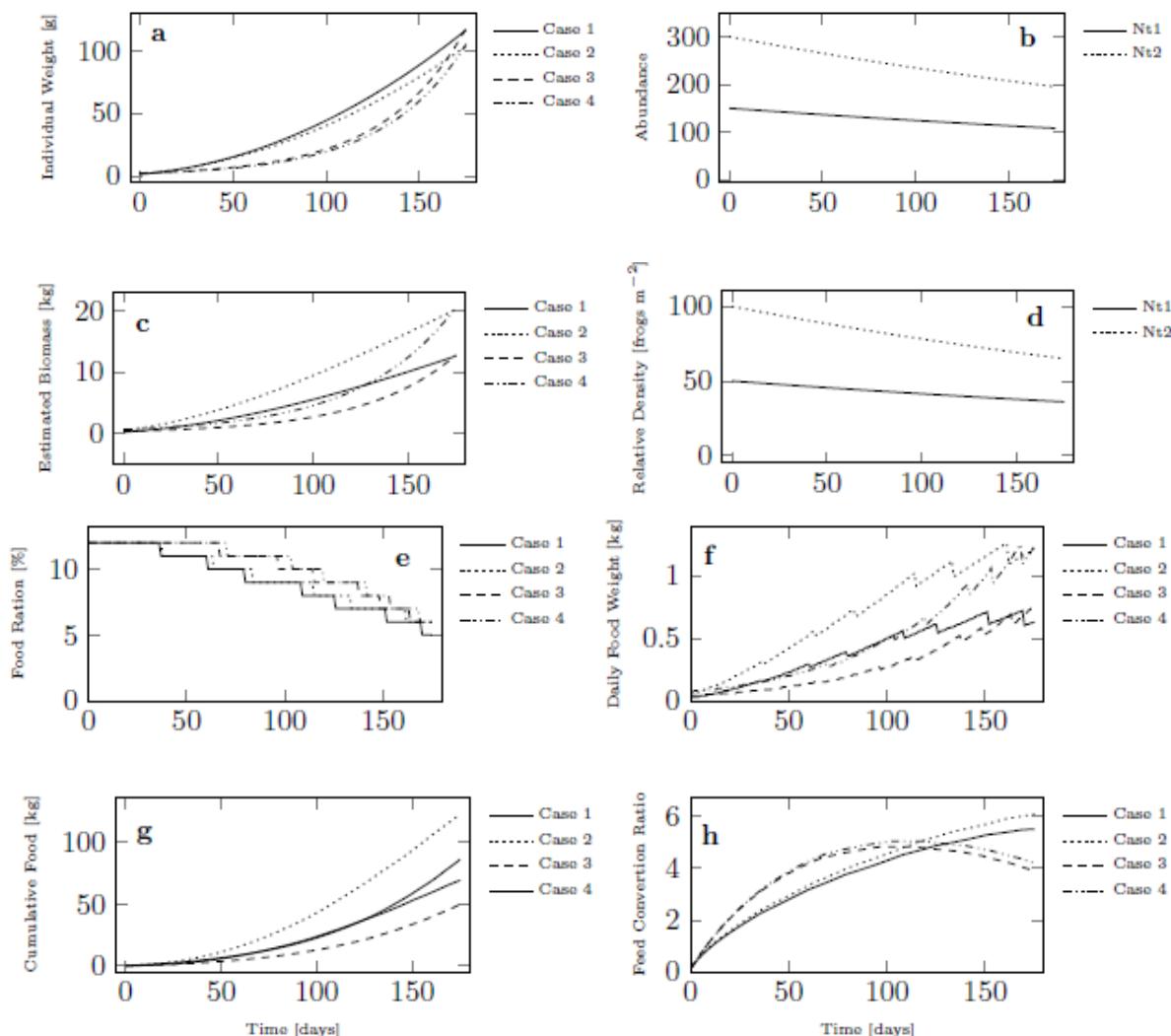
For cumulative food, the tendency in case 1 and 2 shows an expo-linear performance (Fig. 1g), but in cases 3 and 4 the exponential behavior remain. All graphics shows lack of smoothness derived from the effects of changes in food daily ration. The final calculation of cumulative food consumption shows significative differences between cases. For cases 1 and 3, the difference is 77.42 %; and for cases 2 and 4, the difference is 70.03 %. For annual productivity, all the simulated cases calculate close values compared to experimental data.

Relative errors of observations and estimations were also calculated (Table 6). No errors were found in final abundance. Errors in total food consumption were no estimated because no experimental data were reported. For feed conversion ration (FCR), the results reported by Rodríguez- Serna et al. (1996) were based in food intake, but in this work it has been determined by



total food consumption (consumed and not consumed). The higher relative errors were found in initial biomass and initial individual weight for cases 1 and 2. Because the biomass is estimated with the values of individual estimated weight, the errors are similar between independent variables inside the cases.

Figure 1: Simulation results of growth, abundance and food consumption on *Rana catesbeiana* population. Estimations were carried out in four different cases (Table 4) for mean individual growth (a), total abundance (b), total biomass (c), relative density (d), feed daily individual intake (e), feed total daily intake (f), total cumulative food consumption (g) and feed conversion ratio (h).



**Table 1:** General experimental data of Rodríguez-Serna *et al.* (1996)

Parameter	Treatment 1 (50 frogs)	Treatment 2 (100 frogs m ⁻²)
Initial body weight (g)	2.2	2.03
Final body weight (g)	118	105
Survival (%)	72	65
Total culture time (days)	175	175

Table 2: Mean individual weight functions used in the model

Function name	Function type	Weight function
\bar{W}_{1t}	Second order polynomial function	$(1.140578 + 0.0551772 \cdot t)^2$
\bar{W}_{2t}	Second order polynomial function	$(1.22388 + 0.0511682 \cdot t)^2$
\bar{W}_{3t}	Simple exponential growth function	$2.2 \cdot \exp\left[\left(\frac{\ln(118) - \ln(2.2)}{175}\right) \cdot t\right]$
\bar{W}_{4t}	Simple exponential growth function	$2.03 \cdot \exp\left[\left(\frac{\ln(105.5) - \ln(2.03)}{175}\right) \cdot t\right]$

Table 3: Estimated total frog abundance

Function name	Function type	Abundance function
N_{1t}	Simple mortality curve	$150 \cdot \exp\left[\left(\frac{\ln(100) - \ln(72)}{175}\right) \cdot t\right]$
N_{2t}	Simple mortality curve	$300 \cdot \exp\left[\left(\frac{\ln(100) - \ln(65)}{175}\right) \cdot t\right]$

Table 4: Particular cases simulated in this study

Case	Estimated daily food consumption
1	$\bar{W}_{1t} \cdot N_{1t} \cdot f_t$
2	$\bar{W}_{2t} \cdot N_{2t} \cdot f_t$
3	$\bar{W}_{3t} \cdot N_{1t} \cdot f_t$
4	$\bar{W}_{4t} \cdot N_{2t} \cdot f_t$

**Table 5:** Comparison of simulation cases with previous experimental results

Parameter	50 frogs m ⁻²			100 frogs m ⁻²		
	Experiment ^a	Case 1	Case 3	Experiment ^a	Case 2	Case 4
Initial Biomass (kg)	0.330	0.195	0.330	0.609	0.449	0.609
Final Biomass (kg)	12.747	12.589	12.744	20.523	20.201	20.533
Initial individual weight (g)	2.200	1.300	2.200	2.030	1.490	2.030
Final individual weight (g)	118.030	116.566	118.000	105.250	103.598	105.300
Total food consumption (kg)	N/A ^b	69.268	49.775	N/A ^b	122.896	86.129
Final Abundance	108.000	108.000	108.000	195.000	195.000	195.000
FCR	1.690 ^c	5.500	3.900	1.670 ^c	6.080	4.190
Estimated productivity	9.030	8.752	8.860	14.320	14.044	14.275
(kg m ⁻² year ⁻¹)						

^a Experimental data from Rodríguez-Serna et al. (1996)^b N/A: Not available^c Reported only consumed food**Table 6:** Percentage of relative error between observed and estimated results.

Parameter	50 frogs m ⁻²		100 frogs m ⁻²	
	Case 1	Case 3	Case 2	Case 4
Initial Biomass	40.10	0	26.27	0
Final Biomass	1.23	0.025	1.568	0.048
Initial individual weight	40.90	0	26.6	0
Final individual weight	1.24	0.02	1.56	0.04
Estimated productivity	3.07	1.88	1.92	0.31

Discussion

The model construction was done taking into account basic definitions of production by Sparre & Venema (1998), Alagaraja (1991) and Pitcher & Hart (1982). Its main idea was to make a simple, non-dynamic model that can be programmed in non-specialized modeling package (like Excel©). For this reason, it was conceptualized in discrete time to diminished complexity or numerical integration algorithms. Nevertheless, the general functions used to determine individual weight W⁻ t, mortality N_t and food ration f_t can be selected by the modeler. In this case, we selected a set of empirical and theoretical equations as examples in order to analyze the model performance in particular cases. The work of Rodríguez-Serna et al. (1996) was selected because their methods can be easily replicated in silico and their experimental results can be used as empirical validation. The estimations of individual growth weight in case 1 and 2 were made by calculation derived of experimental data (Table 5). This cases corresponded to an actual measured growth performance. On



the other hand, in cases 3 and 4 a simple exponential growth was assumed. As a result, the growth curves have similar values at the beginning and the end of the experiment, but the dynamic is very different. This performance can be explained because the derivatives of the second order polynomial equations used in case 1 and 2 are linear, meanwhile the rate of change in the equations used in case 3 and 4 are exponential. As a consequence, the domains of all the growth equations used in this study must be used only for a short period of time (e.g.175 days), because they are divergent functions, and they tend to increase without restrictions.

When the results of the modelation are compared with actual experimental data have inherent differences caused by the multiple factors that influence growth and yield. Neveu (2009) pointed out the difficult to make comparisons within the farming of neotropical species, like *R. catesbeiana*, *R. ridibunda* and *R. esculenta*. In fact, there seems to be a lot of variation among experimental data for weight, surveillance and food consumption. For example, Olvera-Novoa et al. (2007)

cultured frogs under six different treatments to assess the impact growth changes in response to protein feed content (20, 28, 40, 42, 50 and 58 %). The density was set in 100 individuals m⁻² and the feed ratio was fixed to 6% of the body weight per day. The results showed increments in 24.19, 36.32, 42.91, 46.88, 42.97 and 49.93 grams respectively, after 49 days of culture. According to the results of this study, only similar estimations were presented between the treatment of 20% of protein content and case 1. For the other cases, the model tends to subestimate mean individual weight, specially in cases 3 and 4. In other experiment, Fenerick & Verardino (2005) studied the performance of bullfrog fed with 4 treatments of commercial food during 125 days. Its results shows increments of 90.68, 176.3, 183.54 and 196.9 g during summer time in Brasil. Similar results are predicted only for the first treatment (case 1, 90.30 g from day 37 to day 161; case 3, 90.55 g from day 42 to day 166 and case 4, 90.48 g from day 47 to day 171).

On the other hand, for surveillance estimations, this work assumes only natural mortality under a classic model of exponential mortality (Sparre & Venema, 1998). However, literature reports a wide range of mortalities by multiple causes. Rodríguez-Serna et al. (1996) reported mortalities of 100 % after 8 culture weeks under high stocking densities (200 froglets m⁻²). The cause was red leg disease in a vertical intensive culture system. They also mentioned observed mortalities from 42 to 66% in a period of 92 culture days. These values are reported for brazilian commercial frog farms. In other study, Olvera-Novoa et al. (2007) reported mortalities from 39.66% to 0% after 49 culture days. For other species, Neveu (2009) measured survival rates from 14.8 % to 77.4% after years of culture in different phenotypes of the esculenta hybridogenetic complex.

In food consumption, the model predict values of FCR from 3.9 to 6.08. The model considered both consumed and not consumed food. However, this predicted values are 2 or 3 times the values obtained under experimentation. Olvera-Novoa et al. (2007) calculated FCR based only on eaten food, and they obtained values from 0.99 to 1.52. (Fenerick & Verardino, 2005) mentioned in their discussion FCR values from 0.9 to 2.5. They also pointed out the need to be careful to make general assumptions when different experiments are compared each other, due to the great variety of environmental conditions among experimental procedures. However, the results obtained in the simulations of case 1 and 2 suggest a high waste of feed in experimental procedures.

Conclusions

Particular cases of the general model make closer predictions in cases 1 and 2 for individual mean weight and surveillance. However, predictions for FCR indicated more than three times food



wasted compared with experimental procedures under a simulation time of 175 days. Because the model is algebraic, it can be easily implemented in many computational packages. It could serve as an useful tool to predict the daily and total food consumption under different scenarios, helping to estimate total feed frog consumption and helping to assess feed total cost to the manager.

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